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FORM FACTORS FOR PARALLELEPIPEDS

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Аннотация—В статье представлены таблицы двух основных угловых коэффициентов (φ_{AB}, φ_{AC}) для граней параллелепипеда, заполненного однородной серой средой. Достаточно подробные и точные такие таблицы обеспечивают алгебраическое вычисление угловых коэффициентов всех типов (F–F, F–V, V–V) для граней и объемов системы зон в виде ортогональных параллелепипедов. Для φ_{AB} дается простая формула (6) с небольшой погрешностью при определенных значениях агрументов. Погрешность вычислений по точным формулам (3) и (5) оценивается менее процента. Результаты дополняют имеющиеся литературные данные [1, 2]. Они необходимы для зональных расчетов переноса лучистой энергии в инженерной и физической теплотехнике.

NOMENCLATURE

F_A, F_B, F_C,	surface zones [m^2];
dF_A, dF_B, dF_C,	surface zone elements;
$x, y, z,$ $x_1, y_1, z_1,$ $x_2, y_2, z_2,$	coordinates of dF_A, dF_B, dF_C;
F, v,	indices of the surface and volume zones;
l_1,	beam length between dF_A and dF_B, [m];
l_2,	beam length between dF_A and dF_C;
H,	altitude, see Fig. 1 [m];
θ,	angle between the beam direction (l_0) and the normal to the surface element;
$\theta_1, \theta_2, \theta,$ $\alpha_1, \alpha_2,$ $\beta_1, \beta_2,$	angles, see Fig. 1;
φ_{AB}, φ_{AC},	the form factors for the two surfaces A and B, or A and C [dimensionless];
k,	absorption coefficient of the medium [m^{-1}];
L_0, L_*,	the sides of the rectangular surface A (ribs of the parallelepiped), see Fig. 1;
$u = kL_*$,	dimensionless rib;
l,	$= L_0/L_*$;
h,	$= H/L_*$;
Z_*,	$= Z/L_*$;
x_*,	$= X/L_*$;
$\varphi(u)$,	the radiation fraction leaving gas cube with rib u;
φ^0_{AB},	$\equiv (\varphi_{AB})_{k=0}$.

THE ZONE method is widely used in practice of radiant heat-transfer calculations. Its advantage is in the clear physical interpretation of all quantities. The main difficulties appear in the computation of the form factors. Hottel and Cohen [1] have solved this problem for cubic zones with orthogonal sides. The results are presented graphically for form factors of the type F–F, F–V and V–V. The main disadvantage of the work just mentioned is low accuracy of the results due to graphical integration and representation. Besides, the cubic geometry is of lesser interest than the parallelepiped; but the results for parallelepiped zones would have been much more complicated had they been represented in the same way. The other way of solving this problem consists in tabulating the simplest basic form factors φ_{AB} and φ_{AC} (Fig. 1). The quantity $\varphi_{AB(C)}$ is the probability of the fact that photon radiated by the surface A hits directly the surface $B(C)$ without interaction with the medium. It can be easily shown that sufficiently detailed tables of φ_{AB} and φ_{AC} provide for algebraic calculation of the form factors for any pair of surface and volume zones because of distributivity and reciprocity. Thus six- and five-fold integrals for form factors of the type V–V and V–F are expressed by F–F four-fold integrals. The values of φ_{AB} and φ_{AC} are tabulated in the present paper.

The disadvantage of this procedure in comparison with the method cited [1] is that the elementary calculations appear to be very long in some cases and one can meet subtractions of approximately equal values. Then the demand for the volume and accuracy of tables φ_{AB} and φ_{AC} is too high. Hence, both methods supplement each other. We continued our previous work [2] where the tables of the two-dimensional form factors φ_{AB} and φ_{AC} had been presented. They should be added to the tables given here.

The total formula of the form factor for two surfaces A and B is given as

$$\varphi_{AB} = \frac{1}{\pi F_A} \int_{F_A} \mathrm{d}F_A \int_{F_B} \exp(-kl) \cos\theta_A \cos\theta_B \,.\, l^{-2}\, \mathrm{d}F_B. \tag{1}$$

For our form factor φ_{AB} we have (see Fig. 1)

$$\begin{aligned} &\cos\theta_A = \cos\theta_B = \cos\theta_1 = \cos\alpha_1 \,.\, \cos\beta_1\,;\; l_0 \equiv l_1 = \frac{H}{\cos\alpha_1 \,.\, \cos\beta_1} \\ &\mathrm{d}F_A = \mathrm{d}z \,.\, \mathrm{d}x; \qquad \mathrm{d}F_B = \mathrm{d}x_1 \,.\, \mathrm{d}z_1; \qquad F_A = L_0 L_*; \\ &\mathrm{d}x = \frac{H}{\cos^2\beta_1}\mathrm{d}\beta_1; \qquad \mathrm{d}z_1 = \frac{H}{\cos^2\alpha_1 \,.\, \cos\beta_1}\mathrm{d}\alpha_1. \end{aligned} \tag{2}$$

Using (1) and (2) we find that

$$\varphi_{AB} = \frac{4}{\pi L_0 L_*} \int_0^{L_0} \mathrm{d}z \int_0^{L_*} \mathrm{d}x \int_0^{\arctan (L_* - x)/H} \cos\beta_1 \,\mathrm{d}\beta_1 \int_0^{\arctan[(L_0 - z)/H \,.\, \cos\beta_1]} \exp\left(-\frac{kH}{\cos\alpha_1 \cos\beta_1}\right) \cos^2\alpha_1 \,.\, \mathrm{d}\alpha_1$$

or in dimensionless form

$$\varphi_{AB}(u, h, l) = \frac{4}{\pi l} \int_0^{l} \mathrm{d}z_* \int_0^{1} \mathrm{d}x_* \int_0^{\arctan (1 x_*)/h} \cos\beta_1 \,\mathrm{d}\beta_1 \int_0^{\arctan[(l - z_*)/h \cos\beta_1]} \exp\left(-\frac{uh}{\cos\alpha_1 \,.\, \cos\beta_1}\right) \cos^2\alpha_1 \,\mathrm{d}\alpha_1. \tag{3}$$

For the form factor φ_{AC} we have (see Fig. 1)

$$\cos\theta_A \equiv \cos\theta_2 = \cos\alpha_2 \,.\, \cos\beta_2; \qquad \cos\theta_B \equiv \cos\theta = \cos\alpha_2 \,.\, \sin\beta_2; \qquad dF_C = dy_2\, dz_2$$

$$dy_2 = -(L_* - x)\frac{d\beta_2}{\sin^2\beta_2}; \qquad dz_2 = \frac{L_* - x}{\sin\beta_2 \,.\, \cos^2\alpha_2} d\alpha_2; \qquad l_0 \equiv l_2 = \frac{L_* - x}{\sin\beta_2 \,.\, \cos\alpha_2}. \tag{4}$$

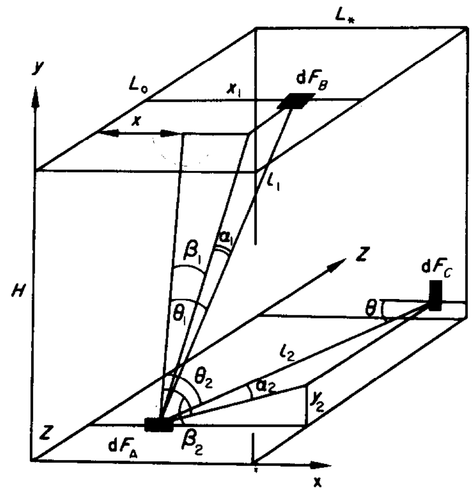

FIG. 1. Diagram for the derivation of formulae (3) and (5).

Using (1) and (4) we find that

$$\varphi_{AC} = \frac{2}{\pi L_0 L_*}\int_0^{L_0} dz \int_0^{L_*} dx \int_{\arctan(L_*-x)/H}^{\pi/2} \cos\beta_2\, d\beta_2 \int_0^{\arctan[(L_0-z)/(1-x)\sin\beta_2)} \exp\left[-\frac{k(L_*-x)}{\cos\alpha_2 \,.\, \sin\beta_2}\right]\cos^2\alpha_2\, d\alpha_2$$

or in a dimensionless form

$$\varphi_{AC}(u, h, l) = \frac{2}{\pi l}\int_0^{l} dz_* \int_0^{1} dx_* \int_{\arctan(1-x_*)/h}^{\pi/2} \cos\beta_2\, d\beta_2 \int_0^{\arctan[(l-z_*)/(1-x_*)\sin\beta_2]} \exp\left[-\frac{u(1-x_*)}{\cos\alpha_2 \,.\, \sin\beta_2}\right]\cos^2\alpha_2 \,.\, d\alpha_2. \tag{5}$$

The form factors φ_{AB} and φ_{AC} are given in Tables 1 and 2. They have been calculated by formulae (3) and (5), the Gaussian quadrature with five abscissas being used.

Table 1. *Form factor* φ_{AB} *(only decimal figures are presented)*

h	l									
	0·1	0·3	0·5	0·7	1·0	1·5	2·0	3·0	5·0	10·0
$u = 0·01$										
0·1	3854	6642	7513	7917	8237	8513	8669	8833	8951	9008
0·3	1315	3319	4483	5171	5772	6286	6552	6829	7083	7303
0·5	0693	1923	2842	3489	4127	4726	5056	5400	5687	5946
0·7	0431	1236	1911	2442	3019	3612	3960	4337	4652	4914
1·0	0247	0723	1154	1527	1976	2494	2825	3208	3545	3810
1·5	0123	0364	0594	0807	1090	1461	1730	2074	2406	2677
2·0	0072	0215	0354	0487	0672	0934	1141	1432	1739	2008
3·0	0033	0099	0164	0228	0320	0461	0585	0782	1026	1274
5·0	0012	0036	0060	0083	0118	0174	0227	0323	0469	0659
10·0	0003	0009	0014	0020	0029	0043	0057	0084	0133	0224
$u = 0·05$										
0·1	3834	6604	7467	7867	8184	8457	8611	8773	8890	8946
0·3	1296	3271	4415	5089	5678	6180	6440	6710	6957	7171
0·5	0678	1880	2776	3406	4025	4605	4924	5256	5531	5781
0·7	0418	1199	1852	2365	2922	3491	3824	4184	4483	4732
1·0	0236	0693	1106	1462	1890	2382	2696	3056	3371	3617
1·5	0115	0342	0558	0758	1023	1369	1619	1937	2240	2485
2·0	0067	0198	0327	0449	0618	0858	1048	1311	1585	1823
3·0	0029	0088	0145	0202	0283	0408	0516	0688	0898	1107
5·0	0010	0029	0049	0068	0096	0142	0186	0263	0379	0527
10·0	0002	0006	0010	0013	0019	0029	0038	0056	0088	0147
$u = 0·1$										
0·1	3810	6556	7410	7805	8118	8387	8539	8700	8814	8870
0·3	1274	3211	4331	4989	5563	6050	6302	6563	6803	7011
0·5	0659	1826	2695	3305	3902	4459	4765	5080	5343	5581
0·7	0402	1154	1781	2273	2804	3346	3661	4001	4281	4514
1·0	0224	0657	1047	1385	1788	2250	2542	2876	3167	3392
1·5	0107	0317	0516	0701	0944	1262	1490	1777	2049	2266
2·0	0060	0179	0295	0405	0557	0773	0941	1174	1412	1616
3·0	0025	0075	0125	0173	0243	0349	0442	0587	0761	0930
5·0	0008	0023	0038	0053	0075	0111	0144	0203	0291	0399
10·0	0001	0003	0006	0008	0012	0017	0023	0034	0053	0087
$u = 0·2$										
0·1	3760	6461	7277	7683	7988	8250	8399	8555	8666	8719
0·3	1230	3095	4167	4796	5340	5800	6037	6281	6508	6702
0·5	0623	1726	2542	3112	3667	4181	4462	4749	4988	5206
0·7	0373	1068	1647	2099	2584	3073	3357	3659	3906	4113
1·0	0202	0591	0941	1242	1601	2007	2262	2550	2796	2986
1·5	0091	0271	0442	0599	0806	1072	1262	1498	1716	1886
2·0	0049	0146	0240	0330	0453	0626	0760	0941	1124	1274
3·0	0019	0056	0092	0128	0179	0257	0324	0427	0547	0659
5·0	0005	0014	0023	0032	0045	0067	0087	0122	0172	0230
10·0	0	0001	0002	0003	0004	0006	0008	0012	0019	0030

Table 1—continued

$u = 0{\cdot}5$										
h	l									
	0·1	0·3	0·5	0·7	1·0	1·5	2·0	3·0	5·0	10·0
0·1	3617	6187	6971	7331	7612	7857	7994	8139	8241	8287
0·3	1107	2774	3717	4261	4727	5115	5313	5516	5706	5870
0·5	0527	1455	2133	2599	3046	3452	3670	3890	4071	4239
0·7	0297	0849	1303	1653	2022	2386	2592	2807	2980	3126
1·0	0147	0430	0683	0897	1149	1425	1595	1781	1934	2053
1·5	0058	0170	0277	0374	0501	0659	0769	0899	1014	1101
2·0	0027	0079	0130	0178	0243	0333	0400	0487	0569	0634
3·0	0008	0022	0037	0051	0072	0102	0127	0165	0205	0239
5·0	0001	0003	0005	0007	0010	0015	0019	0026	0036	0045
$u = 1{\cdot}0$										
0·1	3392	5758	6465	6785	7035	7252	7375	7503	7592	7630
0·3	0930	2313	3076	3506	3868	4163	4311	4464	4609	4733
0·5	0399	1096	1595	1929	2242	2518	2663	2807	2925	3039
0·7	0203	0579	0883	1112	1347	1571	1693	1819	1917	2004
1·0	0087	0253	0400	0522	0662	0809	0896	0988	1062	1119
1·5	0027	0079	0127	0171	0227	0294	0338	0388	0430	0460
2·0	0010	0029	0047	0064	0086	0116	0138	0164	0187	0204
3·0	0002	0005	0008	0011	0016	0022	0027	0034	0041	0046
$u = 2{\cdot}0$										
0·1	2986	4998	5576	5832	6032	6206	6306	6409	6478	6504
0·3	0659	1615	2117	2389	2610	2786	2873	2964	3053	3127
0·5	0230	0625	0896	1070	1225	1357	1423	1489	1544	1600
0·7	0096	0271	0408	0506	0603	0689	0735	0781	0816	0850
1·0	0030	0088	0138	0178	0221	0264	0287	0312	0330	0346
1·5	0006	0017	0027	0036	0047	0059	0066	0074	0080	0085
2·0	0001	0004	0006	0008	0011	0014	0017	0019	0021	0023
$u = 5{\cdot}0$										
0·1	2053	3310	3639	3781	3891	3991	4049	4107	4141	4150
0·3	0239	0562	0711	0785	0842	0886	0907	0931	0956	0974
0·5	0045	0119	0164	0190	0211	0228	0236	0244	0252	0260
0·7	0010	0028	0041	0049	0057	0063	0066	0069	0071	0074
1·0	0001	0004	0006	0007	0009	0010	0010	0011	0011	0012

Table 2. Form factor φ_{AC} (only decimal figures are presented)

h	l									
	0·1	0·3	0·5	0·7	1·0	1·5	2·0	3·0	5·0	10·0
$u = 0{\cdot}01$										
0·1	0240	0359	0399	0419	0437	0452	0460	0467	0473	0477
0·3	0397	0725	0878	0968	1048	1119	1157	1197	1232	1259
0·5	0466	0915	1158	1312	1456	1590	1663	1741	1807	1862
0·7	0505	1027	1331	1533	1731	1921	2028	2142	2239	2320
1·0	0538	1123	1484	1735	1990	2246	2394	2557	2696	2811
1·5	0563	1198	1606	1900	2211	2535	2733	2955	3151	3310
2·0	0575	1232	1662	1975	2314	2678	2905	3168	3404	3598
3·0	0584	1259	1707	2038	2402	2802	3059	3368	3657	3900
5·0	0589	1274	1732	2073	2451	2874	3153	3498	3835	4134
10·0	0591	1280	1743	2088	2473	2907	3196	3560	3932	4281
$u = 0{\cdot}05$										
0·1	0239	0357	0396	0417	0434	0449	0457	0464	0470	0474
0·3	0394	0718	0869	0957	1035	1104	1142	1181	1214	1240
0·5	0461	0904	1142	1292	1433	1563	1634	1709	1772	1825
0·7	0499	1012	1310	1506	1699	1882	1985	2095	2188	2265
1·0	0531	1104	1456	1699	1947	2193	2335	2491	2623	2732
1·5	0555	1175	1572	1856	2155	2466	2654	2865	3049	3199
2·0	0566	1207	1623	1926	2251	2598	2813	3061	3282	3462
3·0	0574	1231	1664	1982	2330	2710	2953	3241	3508	3732
5·0	0578	1244	1686	2013	2373	2772	3033	3352	3660	3928
10·0	0580	1249	1694	2024	2389	2797	3066	3399	3733	4039
$u = 0{\cdot}1$										
0·1	0237	0355	0394	0413	0431	0445	0453	0460	0466	0470
0·3	0390	0710	0857	0943	1019	1087	1123	1160	1192	1218
0·5	0456	0890	1122	1268	1404	1529	1598	1670	1730	1781
0·7	0492	0994	1283	1474	1659	1835	1933	2038	2126	2199
1·0	0522	1082	1423	1657	1894	2129	2264	2411	2536	2637
1·5	0545	1148	1531	1803	2088	2383	2560	2758	2929	3067
2·0	0555	1176	1577	1867	2176	2503	2705	2935	3138	3304
3·0	0562	1198	1613	1916	2245	2601	2827	3093	3336	3537
5·0	0565	1209	1631	1941	2281	2653	2893	3183	3459	3695
10·0	0567	1212	1637	1950	2292	2670	2916	3217	3511	3773
$u = 0{\cdot}2$										
0·1	0235	0351	0388	0407	0424	0438	0445	0452	0458	0461
0·3	0383	0693	0835	0917	0989	1052	1086	1121	1151	1174
0·5	0445	0863	1085	1222	1350	1466	1529	1595	1651	1697
0·7	0479	0960	1233	1411	1584	1746	1836	1931	2010	2076
1·0	0506	1039	1359	1576	1795	2009	2131	2263	2373	2463
1·5	0526	1097	1453	1703	1963	2229	2386	2560	2708	2828
2·0	0534	1120	1491	1756	2036	2328	2505	2705	2879	3019
3·0	0539	1137	1519	1795	2090	2404	2600	2826	3028	3193
5·0	0542	1144	1531	1812	2114	2439	2644	2887	3110	3297
10·0	0542	1146	1534	1816	2120	2447	2655	2903	3135	3334

Table 2—continued

h	l: 0·1	0·3	0·5	0·7	1·0	1·5	2·0	3·0	5·0	10·0
					$u = 0{\cdot}5$					
0·1	0228	0338	0373	0390	0405	0418	0424	0430	0435	0438
0·3	0364	0648	0774	0845	0906	0960	0987	1016	1041	1059
0·5	0416	0791	0982	1098	1204	1298	1348	1400	1443	1479
0·7	0443	0867	1099	1246	1386	1513	1583	1655	1714	1764
1·0	0463	0925	1191	1367	1539	1702	1793	1889	1968	2033
1·5	0476	0963	1253	1450	1649	1845	1957	2077	2178	2258
2·0	0480	0977	1275	1480	1690	1900	2023	2157	2269	2359
3·0	0483	0984	1288	1499	1716	1936	2067	2211	2335	2434
5·0	0484	0987	1292	1504	1723	1946	2080	2229	2359	2463
10·0	0484	0987	1292	1504	1723	1947	2081	2231	2361	2466
					$u = 1{\cdot}0$					
0·1	0218	0319	0349	0365	0377	0388	0393	0399	0402	0404
0·3	0335	0582	0686	0743	0791	0832	0852	0874	0892	0906
0·5	0374	0689	0841	0930	1008	1076	1111	1147	1177	1202
0·7	0393	0741	0920	1028	1128	1216	1262	1310	1349	1382
1·0	0405	0776	0975	1100	1218	1326	1383	1442	1491	1532
1·5	0411	0795	1006	1142	1273	1395	1462	1532	1589	1635
2·0	0413	0800	1015	1154	1289	1416	1487	1561	1622	1671
3·0	0414	0803	1018	1159	1296	1426	1499	1576	1639	1690
5·0	0414	0803	1018	1159	1297	1428	1501	1578	1642	1694
					$u = 2{\cdot}0$					
0·1	0198	0285	0310	0321	0331	0339	0343	0347	0350	0351
0·3	0287	0478	0551	0589	0620	0646	0659	0672	0683	0691
0·5	0311	0540	0639	0693	0739	0777	0796	0816	0832	0845
0·7	0319	0564	0675	0738	0792	0838	0861	0885	0904	0920
1·0	0324	0577	0695	0764	0824	0876	0902	0929	0951	0969
1·5	0325	0582	0703	0774	0837	0892	0921	0949	0973	0992
2·0	0326	0583	0704	0776	0840	0895	0924	0953	0977	0997
3·0	0326	0583	0704	0776	0840	0896	0925	0954	0978	0998
					$u = 5{\cdot}0$					
0·1	0154	0212	0226	0232	0238	0243	0245	0247	0248	0248
0·3	0196	0297	0327	0342	0353	0362	0367	0372	0375	0377
0·5	0202	0310	0345	0362	0376	0387	0392	0398	0402	0405
0·7	0203	0313	0349	0367	0381	0392	0398	0404	0409	0411
1·0	0203	0313	0350	0368	0382	0394	0400	0406	0410	0413

In our previous work [2] the two-dimensional form factors φ_{AB} and φ_{AC} (here φ_{AB} and φ_{AC} at $l = \infty$) are correct within five units in the fourth decimal place. The present calculations and the previous ones differ by an additional (fourth) integration. Therefore they are also correct within some (up to ten) units in the last decimal place.

The present authors' results are compared with those by Hottel and Cohen [1] in Fig. 2 which shows the radiation fraction leaving the gas cube with a rib u. The present authors' results are presented by a solid line. They are computed by the formula

$$\varphi(u) = \frac{3}{2u}[1 - 4\varphi_{AC}(u, 1, 1) - \varphi_{AB}(u, 1, 1)].$$

The discrepancy in the results is beyond the calculation error.

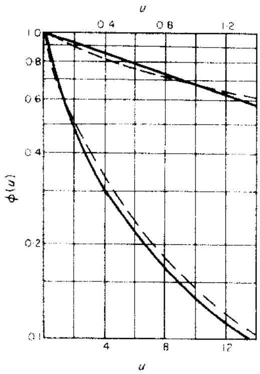

FIG. 2. Radiation fraction leaving gas cube with rib u (Curve 1, the present authors' computations; Curve 2, Hottel and Cohen's data [1]).

The table of φ_{AB} can be expanded as φ_{AB} has the same value for arguments u, h, l_1 and u_2, h_2, l_2

$$\varphi_{AB}(u, h, l) = \varphi^0_{AB}(h, l) \exp\left\{- u\sqrt{\left[h^2 + \tfrac{1}{8}\left(1 + \frac{h^2 l^2}{l^2 + h^2}\right)\right]}\right. \tag{6}$$

which are related by the following equations

$$u_2 = u_1 l_1, \qquad h_2 = \frac{h_1}{l_1}, \qquad l_2 = \frac{1}{l_1}.$$

Formula (6) applies fairly well to the calculation of φ_{AB} for h high enough and small enough l where φ^0_{AB} is expressed by the exact formula

$$\varphi^0_{AB} = \frac{1}{2\pi}\left\{\frac{1}{l}[f\{\sqrt{(h^2 + l^2)}\} - f(h)] + l\left[f\left(\frac{\sqrt{(h^2 + 1)}}{l}\right) - f\left(\frac{h}{l}\right)\right]\right\}$$

$$f(v) = (1 - v)^2 \ln(1 - v^2) + v^2 \ln v^2 + 4v \operatorname{arc\,cot} v.$$

The value

$$u\sqrt{\left[h^2 + \frac{1}{8}\left(1 + \frac{h^2 l^2}{h^2 + l^2}\right)\right]}$$

is the equivalent dimensionless path between the elements dF_A and dF_B. A more suitable form may be chosen. The values of $\varphi^{(3)}_{AB}$ and $\varphi^{(6)}_{AB}$, computed by formulae (3) and (6), are compared in Table 3. The discrepancy increases with the increase of u or l, other arguments being constant.

Table 3. $\varphi_{AB}^{(3)}$ and $\varphi_{AB}^{(6)}$ predicted by formulae (3) and (6) (only decimal figures are presented)

l	$h = 3,\ u = 0{\cdot}01$		$h = 3,\ u = 0{\cdot}1$		$h = 3,\ u = 1{\cdot}0$	
	$\varphi_{AB}^{(3)}$	$\varphi_{AB}^{(6)}$	$\varphi_{AB}^{(3)}$	$\varphi_{AB}^{(6)}$	$\varphi_{AB}^{(3)}$	$\varphi_{AB}^{(6)}$
0·1	003311	003311	002521	002523	0001656	0001664
0·3	009904	009904	007541	007545	0004941	0004969
0·5	01641	01641	01249	01250	0008154	0008208
0·7	02278	02278	01732	01734	001125	001134
1·0	03198	03198	02430	02433	001560	001578
1·5	04610	04611	03494	03502	002189	002234
2·0	05848	05850	04419	04435	002693	002780
3·0	07815	07822	05866	05911	003388	003590
5·0	1026	1028	07612	07736	004070	004523
10·0	1274	1278	09304	09592	004609	005447

The table of φ_{AC} can also be expanded on the basis of the exact equality

$$\varphi_{AC}^{*} = h\varphi_{CA} \tag{7}$$

where φ_{CA} is a tabulated form factor φ_{AC} with arguments for the surface c. In Table 4 some tabulated values of φ_{AC} and calculated values of φ_{AC}^{*} are compared. The difference is sometimes large enough for high h which is the result of the computation errors.

The integral of the general correlation formula for form factors includes the squared distance between the surface elements in the denominator. With the short distance between the surface elements a singularity arises, which by the change to angular coefficients, is smoothed down but does not vanish completely. For the coefficients φ_{AB} the singularity appears at small h and becomes

Table 4. Comparison of tabulated values of φ_{AC} and predicted values of φ_{AC}^{*} [formula (7)]

π/π	u_1, h_1, l_1	φ_{AC}	u_2, h_2, l_2	φ_{CA}	φ_{AC}^{*}	$\varphi_{AC} - \varphi_{AC}^{*}$
1	0·1 2·0 1·0	0·2176	0·2 0·5 0·5	0·1085	0·2170	0·0006
2	0·2 10 1	0·2120	2 0·1 0·1	0·0198	0·1980	0·0140
3	0·05 10 5	0·3733	0·5 0·1 0·5	0·0373	0·3730	0·0003
4	0·01 10 3	0·3560	0·1 0·1 0·3	0·0355	0·3550	0·0010
5	0·5 10 5	0·2361	5 0·1 0·5	0·0226	0·2260	0·0101
6	2 0·1 1	0·0331	0·2 10 10	0·3334	0·0333	−0·0002

stronger with decreasing l and increasing u. Therefore the error in φ_{AB} is highest at $h = 0{\cdot}1$ and $l = 0{\cdot}1$ but it decreases rapidly with the increase of h. For all φ_{AC} the singularity appears for elements in the vicinity of their interface and affects to some extent all numerical calculations. Therefore additional studies of the integral φ_{AC} are carried out. In Table 5 the values of φ_{AC} are summarized for various numbers of knods in the Gauss quadrature. One can see that the integral singularity is strongest at small l and large u that is in accordance with the general analysis. With increasing l, the error decreases rapidly. With small u and moderate l, the values of φ_{AC} coincide for all quadratures. With large u our tabulated φ_{AC} are underestimated throughout.

Table 5. Relation between coefficients φ_{AC} and the number of knods of Gaussian quadrature at $h = 1$. Only decimal figures are presented

u	0·01			5		
number of knods	l					
	0·05	0·6	10	0·05	0·6	10
5	03072	16200	28105	01289	03602	04133
6	03177	16204	28078	01408	03656	04175
8	03260	16207	28001	01509	03694	04198

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Abstract In the present paper, tables are presented for two main form factors (φ_{AB}, φ_{AC}) for the surfaces of a parallelepiped filled with a homogeneous grey fluid. Such tables being detailed and accurate provide for algebraic calculations of form factors of all types (F–F, F–V, V–V) for the surfaces and volumes of a system consisting of orthogonal parallelepipeds. φ_{AB} can be determined by a simple formula (6) with a small error for certain values of the arguments. The errors produced by formulae (3) and (5) are below 1 per cent. The present results supplement data previously reported [1, 2]. They are necessary for zone computation of radiant energy transfer widely used in thermal engineering and thermal physics.

Résumé—On présente dans cet article des tableaux de deux facteurs principaux (φ_{AB}, φ_{AC}) pour les surfaces d'un parallélipipède rempli d'un fluide gris homogène. De tels tableaux détaillés et précis fournissent des calculs algébriques des facteurs de forme de tous types (F–F, F–V, V–V) pour les surfaces et les volumes d'un système consistant en parallélipipèdes orthogonaux. φ_{AB} peut être déterminé par une formule simple (6), avec une faible erreur pour certaines valeurs de l'argument. Les erreurs dues à l'emploi des formules (3) et (5) sont inférieures à 1 pour cent. Les résultats actuels complètent ceux publiés auparavant [1, 2]. Ils sont nécessaires pour le calcul par zones du transport d'énergie de rayonnement, qui est beaucoup employé dans la technique et la physique thermique.

Zusammenfassung—Es werden Tabellen zweier Hauptfaktoren (φ_{AB}, φ_{AC}) wiedergegeben für Oberflächen eines Parallelepipeds, das mit einer homogenen grauen Flüssigkeit gefüllt ist. Diese genauen und ins einzelne gehenden Tabellen ermöglichen die Berechnung von Winkelverhältnissen aller Typen (F–F, F–V, V–V) für die Oberflächen und Volumen eines Systems aus orthogonalen Parallelepipeden. Der Faktor φ_{AB} kann durch eine einfache Formel (6) bestimmt werden mit einem keleinen Fehler bei gewissen des Arguments. Die sich bei Verwendung der Formeln (3) und (5) ergebenden Fehler liegen unter 1 %. Die gegenwärtigen Ergebnisse ergänzen kürzlich mitgeteilte Werte [1, 2]. Sie sind notwendig zu der im Wärmeingenieurwesen und in der Wärmephysik weit verbreiteten abschnittsweisen Berechnung des Energietransports durch Strahlung.